



Mark Scheme

Summer 2023

Pearson Edexcel GCE

Advanced Subsidiary Level

Further Mathematics (8FM0)

Paper 21 Further Pure Mathematics 1

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 40.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.  
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
<b>1 (a)</b>	E.g. $(x \neq 2 \Rightarrow) 5x = 12(x - 2) \Rightarrow x = \dots$ $\frac{5x}{x-2} \geq 12 \Rightarrow 5x \quad x-2 \geq 12 \quad x-2^2 \Rightarrow x-2 \quad 5x-12 \quad x-2 \geq 0 \Rightarrow x = \dots$ or $\frac{5x}{x-2} \geq 12 \Rightarrow \frac{5x-12}{x-2} \geq 0 \Rightarrow x = \dots$ o.e.	M1	2.1
	E.g. $x = 2$ and $(x \neq 2 \Rightarrow) 5x = 12(x - 2) \Rightarrow x = \dots$ $x-2 \quad -7x+24 \geq 0$ or $x-2 \quad 7x-24 \leq 0 \Rightarrow x = \dots, \dots$ or $7x^2 - 38x + 48 \leq 0$ or $-7x^2 + 38x - 48 \geq 0 \Rightarrow x = \dots, \dots$ or $\frac{24-7x}{x-2} \geq 0 \Rightarrow x = \dots, \dots$	dM1	1.1b
	Critical values $x = 2, \frac{24}{7}$	A1	1.1b
	$2 < x \leq \frac{24}{7}$	A1	2.3
		(4)	
<b>(b)</b>	$x = 3$	B1	2.2a
		(1)	
<b>(5 marks)</b>			
<b>Notes:</b>			
<p><b>(a)</b></p> <p><b>M1:</b> For an algebraic method to find the critical value aside from <math>x = 2</math>. May set equal and use <math>x \neq 2</math> or <math>x &lt; 2</math> and <math>x &gt; 2</math> to find the <math>x</math> coordinate of the intersection of line and curve, or may multiply through by <math>x - 2^2</math> and gather terms onto one side and solve the quadratic, or gather all terms onto one side and put over a common denominator and solve the quadratic. Allow with any inequality or equality for the first two marks.</p> <p><b>dM1:</b> Finds <b>both</b> the critical values by a correct algebraic method (allowing for slips rearranging). The <math>x = 2</math> may just be stated or used as a boundary value of the interval. Use of quadratic formula - usual rules. Dependent on previous method mark.</p> <p><b>A1:</b> Correct critical values stated or used in solution. Allow awrt 3.43 for <math>\frac{24}{7}</math> for this mark. A0 if other values also used.</p> <p><b>A1:</b> Deduces the correct inequality. Must be exact. Accept alternative notations, such as <math>\left(2, \frac{24}{7}\right]</math> but formal set notation is not required - score the inequality given. Accept as separate inequalities.</p> <p>If <math>2 &lt; x \leq \frac{24}{7}</math> follows <math>2 \leq x \leq \frac{24}{7}</math> arising from <math>(x-2)(7x-24) \leq 0</math> (oe) then allow A1 b.o.d. that the latter answer is a rejection of the 2 being included.</p>			
<p><b>(b)</b></p> <p><b>B1cao:</b> Deduces the correct value for <math>x</math> and no other values as long as 3 is in their solution set from (a). Allow if the endpoint 2 was included in their answer to (a) as long as it is not given as a solution for (b).</p>			



Question	Scheme	Marks	AOs
<b>2(a)</b>	$3 \cos x - 2 \sin x \Rightarrow 3 \left( \frac{1-t^2}{1+t^2} \right) - 2 \left( \frac{2t}{1+t^2} \right)$	M1	1.1b
	$3 \left( \frac{1-t^2}{1+t^2} \right) - 2 \left( \frac{2t}{1+t^2} \right) = 1 \Rightarrow 3(1-t^2) - 4t = 1+t^2$	M1	1.1b
	$2t^2 + 2t - 1 = 0^*$	A1*	2.1
		<b>(3)</b>	
<b>(b)</b>	$t = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot -1}}{2 \cdot 2} \left\{ = \frac{-1 \pm \sqrt{3}}{2} = -1.366..., 0.366... \right\}$	M1	1.1b
	$\frac{x}{2} = \arctan -1.366... \text{ or } \frac{x}{2} = \arctan 0.366... \text{ and } \Rightarrow x = ...$	dM1	3.1a
	$x = \text{awrt } -107.6^\circ \text{ or awrt } 40.2^\circ$	A1	1.1b
	$x = -107.6^\circ, 40.2^\circ \text{ cao}$	A1	1.1b
		<b>(4)</b>	
<b>(7 marks)</b>			

**Notes:****(a)**

**M1:** Uses at least one of  $\sin x = \frac{2t}{1+t^2}$  or  $\cos x = \frac{1-t^2}{1+t^2}$  to express  $3 \cos x - 2 \sin x$  in terms of  $t$  only.

**M1:** Uses both correct formulae for  $\sin x = \frac{2t}{1+t^2}$  and  $\cos x = \frac{1-t^2}{1+t^2}$  in  $3 \cos x - 2 \sin x$  equates their expression to 1 and multiplies through by  $1+t^2$  to achieve a quadratic expression in  $t$  (there may be slips in coefficients). Alternatively, gather **all** terms to a single fraction over denominator  $1+t^2$ .

**A1\*:** Collects terms to one side and simplifies to achieve the printed answer, with no errors seen.

**(b)**

**Note:** The question says “hence solve”, so there needs to be evidence of use of part (a). Answers only scores no marks. Minimal requirement would be to see the solutions for  $t$  before stating the final answers.

**M1:** Selects a correct process to solve  $2t^2 + 2t - 1 = 0$  (calculator, quadratic formula, completing the square) to obtain at least one value for  $t$ . Attempts by factorisation are M0.

**dM1:** Adopts a correct strategy of taking  $\arctan$ (their value for  $t$ ) and multiplies the result by 2 to obtain at least one value for  $x$  within the range  $-180^\circ < x < 180^\circ$ . Allow slips on, or miscopies of their roots if the process is correct.

**A1:** One correct answer awrt 1 d.p.

**A1:** Both correct answers to 1 d.p. and no incorrect answers in the range  $-180^\circ < x < 180^\circ$





Question	Scheme	Marks	AOs
<b>3(a)</b>	$xy = c^2$ and $x - 2y = c \Rightarrow$ $2y^2 + cy = c^2$ or $\frac{1}{2}x^2 - cx = c^2$	M1	1.1b
	$2y^2 + cy - c^2 = 0 \Rightarrow y = -c, \frac{c}{2} \Rightarrow x = \dots$ or $x^2 - cx - 2c^2 = 0 \Rightarrow x = -c, 2c \Rightarrow y = \dots$	dM1	1.1b
	$-c, -c$ and $\left(2c, \frac{c}{2}\right)$	A1	2.2a
		(3)	
<b>Alt</b>	General point is $\left(ct, \frac{c}{t}\right) \Rightarrow ct - \frac{2c}{t} = c \Rightarrow ct^2 - 2c = ct$	M1	1.1b
	$\Rightarrow t^2 - t - 2 = 0 \Rightarrow t = 2, -1 \Rightarrow \left(2c, \frac{c}{2}\right)$ or $(-c, -c)$	dM1	1.1b
	$-c, -c$ and $\left(2c, \frac{c}{2}\right)$	A1	2.2a
		(3)	
<b>(b)</b>	Midpoint = $\left(\frac{-c + 2c}{2}, \frac{-c + \frac{c}{2}}{2}\right) = \dots$	M1	1.1b
	$x = \frac{c}{2}$ and $y = -\frac{c}{4} \Rightarrow xy = \frac{c}{2} \times -\frac{c}{4}$ leading to $xy = -\frac{c^2}{8}$	A1cso	2.1
		(2)	

**(5 marks)**

Notes:

**(a)**

**M1:** Solve simultaneously  $xy = c^2$  and  $x - 2y = c$  leading to a quadratic in either  $x$  or  $y$  (and  $c$ ). It is a method mark so allow if e.g. there is a miscopy if the intent is clear.

**Alt:** Uses parametric equations and substitutes into  $x - 2y = c$  leading to a quadratic in  $t$  (and  $c$ ).

**dM1:** Dependent on the first method mark. Solves their 3TQ (usual rules) and proceeds to find at least one set of coordinates for either  $P$  or  $Q$ .

**A1:** Deduces the correct coordinates for  $P$  and  $Q$  (need not be named). Accept as  $x = \dots, y = \dots$  as long as the coordinates are clearly paired.

**(b)**

**M1:** Find the midpoint of their  $P$  and  $Q$ , provided that  $P$  and  $Q$  are not symmetric in the  $y$ -axis (ie. midpoint is not  $(0,0)$ ). May be implied by one correct coordinate if no method shown. Simplification is not required.

**A1cso:** Uses the *correct* coordinates of the midpoint from correct work to show that  $xy = -\frac{c^2}{8}$ . Must be an equation, not just the value of  $a$ . There must be no contrary statements.

Question	Scheme	Marks	AOs
<b>4</b>	Identifies that $\theta_0 = 95$ and $t_0 = 0$ The temperature after 5 minutes is required so $h = 2.5$	B1	3.3
	$t_0 = 0, \theta_0 = 95, \left(\frac{d\theta}{dt}\right)_0 = -0.05 \ 95 - 20 = \dots \left\{ = -\frac{15}{4} = -3.75 \right\}$	M1	3.4
	$\theta_1 \approx \theta_0 + h \left(\frac{d\theta}{dt}\right)_0 \approx 95 + 2.5 \times -3.75 = \dots \left\{ \theta_1 = 85.625 = \frac{685}{8} \right\}$	M1	1.1b
	$\left(\frac{d\theta}{dt}\right)_1 = -0.05 \ 85.625 - 20 = \dots \left\{ -3.28125 = -\frac{105}{32} \right\}$ $\theta_2 \approx \theta_1 + h \left(\frac{d\theta}{dt}\right)_1 \approx 85.625 + 2.5 \times -3.28125$	M1	1.1b
	= awrt 77.4	A1	1.1b
	$\theta_2 = 77.4 > 70$ therefore, the teacher will not be able to start to drink the coffee at 1.20pm	B1ft	3.2a
		(6)	
<b>(6 marks)</b>			

**Notes:**

**B1:** Identifies the correct initial conditions and requirements for  $h$ . These may be implicit in their work.

**M1:** Uses the model to evaluate  $\frac{d\theta}{dt}$  at  $t_0$ , using their  $\theta_0$ , allowing for slips if the intent is clear.

**M1:** Applies the approximation formula with their values for  $\theta_0$ ,  $h$ , and  $\left(\frac{d\theta}{dt}\right)_0$  to find a value for  $\theta_1$

**M1:** Attempts to find  $\left(\frac{d\theta}{dt}\right)_1$  with their '85.625' and applies the approximation formula with their values for  $\theta_1$ ,  $h$ , and  $\left(\frac{d\theta}{dt}\right)_1$  to find a value for  $\theta_2$

**A1:** Achieves awrt 77.4

**B1ft:** Obtains a value of  $\theta_2$  in the range  $[40, 100]$ , compares their value of  $\theta_2$  with 70 and draws an appropriate conclusion about whether the teacher will be able to start to drink the cup of coffee at 1.20pm. The conclusion may be minimal, e.g. " $77.4 > 70$  so no" scores B1. Accept "it is too hot" or similar as either a comparison or a conclusion but not both.

E.g. **Acceptable** : " $77.4$  is too hot, so cannot drink the coffee", " $77.4 > 70$ , so too hot"

**Unacceptable** : " $77.4$  is too hot".

Question	Scheme	Marks	AOs
<b>5(a)</b>	A complete method to find the values for $p$ and $q$ . There must be an attempt to find $\overrightarrow{AB} \times \overrightarrow{AC}$ and set equal to a multiple of $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and forms and solve two simultaneous equations for $p$ and $q$	M1	3.1a
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ p & 4 & 6 \\ q & 4 & 5 \end{vmatrix} = 20 - 24\mathbf{i} - 5p - 6q\mathbf{j} + 4p - 4q\mathbf{k}$	M1 A1	1.1b 1.1b
	$\overrightarrow{AB} \times \overrightarrow{AC} = -2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} \quad \text{or} \quad \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \times (\overrightarrow{AB} \times \overrightarrow{AC}) = \mathbf{0}$	B1	2.2a
	$-5p - 6q = -2 \times 3 \text{ and } 4p - 4q = -2 \times 4$ $\{\text{alt : } 32p - 36q = 0, 8p - 8q + 16 = 0, 12q - 10p + 12 = 0\}$	M1	3.1a
	Solves $5p - 6q = 6$ and $4p - 4q = -8$ To find values for $p$ and $q$	dM1	1.1b
	$p = -18, q = -16$	A1	1.1b
		(7)	
<b>Alt</b>	A complete method to find the values for $p$ and $q$ . Attempts scalar product of both $\overrightarrow{AB}$ and $\overrightarrow{AC}$ with $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ sets equal to 0 and solves both equations.	M1	3.1a
	$\begin{pmatrix} p \\ 4 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 2p + 12 + 24 \quad \text{or} \quad \begin{pmatrix} q \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 2q + 12 + 20$	M1 A1	1.1b 1.1b
	$\begin{pmatrix} p \\ 4 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 0 \Rightarrow 2p + 36 = 0 \quad \text{or} \quad \begin{pmatrix} q \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 0 \Rightarrow 2q + 32 = 0$	B1	2.2a
	$\Rightarrow p = \dots \quad \text{or} \quad \Rightarrow q = \dots$	M1	1.1b
	$\Rightarrow p = \dots \quad \text{and} \quad \Rightarrow q = \dots$	dM1	3.1a
	$p = -18, q = -16$	A1	1.1b
		(7)	
<b>(b)</b>	E.g. Area = $\frac{1}{2} \begin{vmatrix} -4 \\ -6 \\ -8 \end{vmatrix} = \frac{1}{2} \left[ \sqrt{-4^2 + -6^2 + -8^2} \right] = \dots$	M1	1.1b
	Area = $\sqrt{29}$ or $\frac{1}{2}\sqrt{116}$	A1	1.1b
		(2)	
<b>(9 marks)</b>			

**Notes:****(a)**

**M1:** A complete method to find the values for  $p$  and  $q$ . There must be an attempt to find  $\vec{AB} \times \vec{AC}$  and sets equal to a **multiple** (not equal to 1) of  $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$  (allowing minor slips) to form and solve two simultaneous equations for  $p$  and  $q$ . This is for the overall approach, so may be scored if the method for the cross product is incorrect, as long as an attempt has been made.

**M1:** A complete method to find the cross product  $\vec{AB} \times \vec{AC}$ . Form should be correct  $(\dots)\mathbf{i} - (\dots)\mathbf{j} + (\dots)\mathbf{k}$  with correct coefficients in each bracket, though allow slips in sign in these.

**A1:** Correct cross product.

**B1:** Deduces that  $\vec{AB} \times \vec{AC} = -2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$  May be implied by their working. Alternatively, a correct cross product statement for the parallel vectors.

**M1:** Forms simultaneous equations by setting the  $\mathbf{j}$  component of the cross product to  $k$  times 3 and setting the  $\mathbf{k}$  component of the cross product to  $k$  times 4, where  $k \neq 1$  is an attempt at a scale factor for the parallel vectors. Alternatively, attempts the cross product and equates at least two components to 0 for form simultaneous equations - the attempt at the cross product may be incorrect for this mark as long as the intent is clear.

**dM1:** Solves their simultaneous equations to find a value for  $p$  or  $q$ . Accept for any  $p$  and  $q$  following forming simultaneous equations (do not be concerned about the method used). Dependent on the previous method mark

**A1:** Correct values for  $p$  and  $q$ .

**Alt**

**M1:** A complete method to find the values for  $p$  and  $q$ . There must be an attempt to apply the scalar product of both  $\vec{AB}$  and  $\vec{AC}$  with  $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$  then set each equal to 0 and solves both equations.

**M1:** A correct method for one of the scalar products.

**A1:** Correct expression for one of the scalar products.

**B1:** Forms at least one correct equation for  $p$  or  $q$  by setting the scalar product equal to zero and producing a correct linear equation for  $p$  or  $q$

**M1:** Proceeds to find at least one of the two values.

**dM1:** Solves to find a value for  $p$  and for  $q$ . Dependent on the previous method mark

**A1:** Correct values for  $p$  and  $q$ .

**(b)**

**M1:** A complete method for the area of the triangle. E.g. may use area =  $\frac{1}{2} |\vec{AB} \times \vec{AC}|$  with their cross product from (a), or starting from scratch, and correct attempt at the modulus. Alternative may find angle between  $\vec{AB}$  and  $\vec{AC}$  using scalar product,  $\cos \theta = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|}$ , and then Area =  $\frac{1}{2} |\vec{AB}| |\vec{AC}| \sin \theta$ .

**A1:** Correct exact area from a correct  $\vec{AB} \times \vec{AC}$  (or correct  $p$  and  $q$  in the alt) (oe). There may be variations in method used, but if an incorrect vectors is used in the process score A0.

Question	Scheme	Marks	AOs
<b>6 (a)</b>	$\frac{dy}{dx} = \frac{2a}{2at}$ or $\frac{dy}{dx} = 2\sqrt{a} \times \frac{1}{2} x^{-\frac{1}{2}}$ or $\frac{dy}{dx} = \frac{2a}{y}$	B1	1.1b
	Finds the perpendicular gradient at sets equal to 2 to find a value of $t$ $-t = 2 \Rightarrow t = -2^*$	M1 A1*	2.1 1.1b
		(3)	
<b>(b)</b>	$t = -2 \Rightarrow 4a, -4a$ Correct equation of the normal $y + 4a = 2x - 4a$	B1	1.1b
	$2x - 12a = \sqrt{4ax} \Rightarrow 2 \cdot 9 - 12a = \sqrt{4a \cdot 9} \Rightarrow 18 - 12a = 6\sqrt{a}$ $\Rightarrow 12a + 6\sqrt{a} - 18 = 0$ or $2x - 12a^2 = 4ax \Rightarrow 2 \cdot 9 - 12a^2 = 4a \cdot 9 \Rightarrow 18 - 12a^2 = 36a$ $\Rightarrow 144a^2 - 468a + 324 = 0$	M1	3.1a
	Solves 3TQ to find a value for $a$ Note: Quadratic for $\sqrt{a}$ leads to $\sqrt{a} = 1$ and $-1.5$ Quadratic for $a$ leads to $a = 1$ and $\frac{9}{4}$	dM1 A1	1.1b 1.1b
	e.g. for $t = -2, a = \frac{9}{4}$ leads to $x = 9$ but $P$ is the other intersection point ( $a = 1$ leads to the correct coordinate of $P(4, -4)$ , or $\sqrt{a}$ cannot be negative) therefore $a = 1$	A1	2.4
		(5)	

(8 marks)

**Notes:****(a)****B1:** any correct expression for  $\frac{dy}{dx}$  , may be unsimplified.**M1:** Finds the perpendicular gradient and sets equal to 2 and uses  $(at^2, 2at)$  to find a value of  $t$ .Alternatively set the gradient equal to  $-\frac{1}{2}$  to find a value of  $t$ . May be seen as part of a longer method finding the equation of the tangent first.**A1\*cs0:** Arrives at correct value  $t = -2$  only from correct work , no errors seen.**(b)****B1:** Correct equation of the normal. May be implied by later working.**M1:** Starts the process of solving simultaneously the equations of the parabola and normal by eliminating  $y$ , substituting  $x = 9$  and forming 3TQ equation for  $\sqrt{a}$  or  $a$  . There may be slips in the process.**dM1:** Dependent on previous mark. Solves their 3TQ to find a value for  $a$ **A1:** Correct possible values for  $a$ **A1:** Deduces the correct value of  $a$  and gives a correct reason.NB: Use of gradient  $-2$  instead of  $2$  in (b) often leads to  $a = 9$ , and can score maximum B0M1M1A0A0.

Alt to (a)	$\frac{dy}{dx} = \frac{2a}{2at}$ or $\frac{dy}{dx} = 2\sqrt{a} \times \frac{1}{2} x^{-\frac{1}{2}}$ or $\frac{dy}{dx} = \frac{2a}{y}$	B1	1.1b
	$t = -2 \Rightarrow y = -4a \Rightarrow \frac{dy}{dx} = \frac{2a}{-4a} = -\frac{1}{2}$ so gradient of normal is $-\frac{1}{-\frac{1}{2}} = 2$ , hence normal parallel to $y = 2x$ (when $t = -2$ )*	M1 A1*	2.1 1.1b
		(3)	

**B1:** As main scheme.

**M1:** Finds the value of the normal when  $t = -2$

**A1:** Correct work to show normal has gradient 2 when  $t = -2$  and a conclusion.

Alt 1 to (b)	$t = -2 \Rightarrow 4a, -4a$	B1	1.1b
	Correct equation of the normal $y + 4a = 2x - 4a$		
	$2x - 12a^2 = 4ax \Rightarrow 4x^2 - 48ax + 144a^2 = 4ax$ $\Rightarrow x^2 - 13ax + 36a^2 = 0$	M1	3.1a
	$\Rightarrow (x - 4a)(x - 9a) = 0 \Rightarrow x = 4a, 9a$	dM1 A1	1.1b 1.1b
	$x = 4a$ is $P$ so other point is $x = 9a$ so $9 = 9a \Rightarrow a = 1$	A1	2.4
		(5)	

(8 marks)

**B1:** Correct equation of the normal.

**M1:** Starts the process of solving simultaneously the equations of the parabola and normal by eliminating  $y$ , and expanding to a quadratic in  $a$  and  $x$ .

**dM1:** Dependent on previous mark. Solves their 3TQ in  $x$  to find the possible values of  $x$  in terms of  $a$

**A1:** Correct values of  $x$  in terms of  $a$

**A1:** Deduces the correct value of  $a$  with a minimal reason or clear correct working shown.

Alt 2 to (b)	$t = -2 \Rightarrow 4a, -4a$	B1	1.1b
	Correct equation of the normal $y + 4a = 2x - 4a$		
	$2at + 4a = 2at^2 - 4a \Rightarrow 2t + 4 = 2t^2 - 8$ $\Rightarrow 2t^2 - 2t - 12 = 0$	M1	3.1a
	$\Rightarrow t^2 - t - 6 = 0 \Rightarrow (t + 2)(t - 3) = 0 \Rightarrow t = "3"$	dM1 A1	1.1b 1.1b
	$t = -2$ is $P$ so other point is when $t = 3$ therefore $9 = a \times 3^2 \Rightarrow a = 1$	A1	2.4
		(5)	

(8 marks)

**B1:** Correct equation of the normal.

**M1:** Substitutes the parametric equations for the parabola into the equation for the normal and cancels the factors  $a$  to produce an equation in  $t$  only.

**dM1:** Dependent on previous mark. Solves their 3TQ in  $t$  to find at least the other value for  $t$

**A1:** Correct other value for  $t$

**A1:** Deduces the correct value of  $a$  with a minimal reason for choice of  $t$  or clear correct working shown.

